# Parallel computation and visualization of three-dimensional, time-dependent, thermal convective flows

Ping Wang and Peggy Li Jet Propulsion Laboratory California Institute of Technology MS 168-522, 4800 Oak Grove Drive Pasadena CA 91109-8099, U.S.A.

#### Abstract

A high-resolution numerical study on parallel systems is reported on three-dimensional, time-dependent, thermal convective flows. Numerical results are obtained for Rayleigh numbers up to  $5 \times 10^7$  and for a Prandtl number 0.733 equivalent to that of air, in a cubical enclosure, which is heated differentially at two vertical side walls. A parallel implementation of the finite volume method with a multigrid scheme is discussed, and a parallel visualization system is developed on distributed systems for visualizing the flow. The details of the three-dimensional, time-dependent flow are described. Separations of the flow near the horizontal walls occur at  $R=10^7$ , and y-variations of the flows are getting stronger with the increases of Rayleigh number.

**Keywords:** parallelization, visualization, thermal convection, finite volume, multigrid

# 1. INTRODUCTION

Natural convection driven by imposed horizontal density gradients finds many applications in engineering: reactor cooling systems, crystal growth procedures, and solar-energy collectors. The most numerically studied form of this problem is the case of a rectangular cavity with differentially heated sidewalls. To date, the two dimensional version of this problem has received considerable attention. However, for the three dimensional case, very few results have been obtained, mainly due to limits in computing power. The significant computational resources of modern, massively parallel supercomputers promise to make such studies feasible. In this paper, a numerical study for 3D, time-dependent thermal convective flow will be investigated. An efficient numerical scheme with a general parallel implementation on massively parallel supercomputers will be presented.

In this paper, we will also discuss our parallel visualization work for presenting numerical results. Visualization of large three-dimensional, time-varying scientific datasets can not be effectively performed using the workstation-based visualization systems due to its demands for large storage and CPU power. Therefore, a parallel rendering system on massively parallel supercomputer is desirable for visualizing this type of dataset. A distributed visualization system, called ParVox, was designed and implemented on the MPP to visualize multiple time-step, 3D scientific datasets in structured grid. In section 5, we will describe the ParVox system and its parallel implementation. In the last section, we will discuss how to use ParVox to visualize the thermal convective flow and present some snap shots of the 3D temperature and velocity fields.

# 2. MATHEMATICAL FORMULATION

The flow domain is a rectangular cavity of 0 < x < l, 0 < y < d, and 0 < z < h. The appropriate governing equations, subject to the Boussinesq approximation, can be written in non-dimensional form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{1}{\sigma} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \nabla^2 u, \tag{2}$$

$$\frac{1}{\sigma} \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \nabla^2 v, \tag{3}$$

$$\frac{1}{\sigma} \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \nabla^2 w + RT, \tag{4}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \nabla^2 T. \tag{5}$$

where  $\sigma$  is the Prandtl number and R is the Rayleigh number. The rigid end walls on x=0,l are maintained at fixed temperatures while the other boundaries are assumed to be insulating.

# 3. NUMERICAL APPROACH

The numerical approach is based on the widely used finite volume method [1] with an efficient and fast elliptic multigrid solver for predicting incompressible fluid flows. This approach has been proven to be a remarkably successful implicit method. A normal, staggered grid configuration is used and the conservation equations are integrated over a macro control volume. Local, flow-oriented, upwind interpolation functions have been used in the scheme to prevent the possibility of unrealistic oscillatory solutions at high Rayleigh numbers. The discretized equations derived from the scheme, including a pressure equation which consumes most of the computation time, are solved by using a parallel multigrid method. This method acts as a conver-

gence accelerator and reduces the CPU time significantly for the entire computation.

The main idea of the multigrid approach is to use the solution on a coarse grid to revise the required solution on a fine grid. In view of this, a hierarchy of grids of different mesh sizes is used to solve the fine grid problem. It has been proven theoretically and practically that the multigrid method has a better rate of convergence [2]. In the present computation, a V-Cycle scheme with a flexible number of grid levels is implemented with Successive Over-Relaxation as the smoother. Injection and linear interpolation are used as restriction operators and interpolation operators respectively.

### 4. PARALLEL IMPLEMENTATION

The finite volume method with multigrid scheme for three-dimensional, time-dependent, thermal convective flows is implemented on distributed memory systems [3]. A flexible parallel code for such flow problems has been designed by using domain decomposition techniques and the MPI communication software. In order to achieve load balancing and to exploit parallelism as much as possible, a general and portable parallel structure based on the domain decomposition techniques is designed for the three dimensional flow domain, which has 1D, 2D and 3D partition features and can be chosen according to different geometries. MPI software is used for the internal communication which is encountered when each subdomain on each processor needs its neighbor's boundary data information. The implementation is carried out on the distributed-memory systems, and the code currently runs on the Intel Paragan, the Cray T3D, the Cray T3E, the IBM SP2, the Beowulf system, and the HP 2000, which can be easily ported to other parallel systems.

### 5. PARALLEL VISUALIZATION

A distributed visualization system, called ParVox, is used to visualize the multiple timestep 3D thermal convective flow on the MPP. ParVox is a parallel volume rendering system designed for visualizing large volumes of time-varying, three-dimensional scientific datasets. It is a parallel implementation of the forward-feeding splatting algorithm [4] utilizing both the object-space decomposition and image-space decomposition. Currently, it is running on the Cray T3D and the Cray T3E using Cray's shmem library for interprocessor communication. We are in the process of porting it to the MPI2 using the newly defined one-sided communication API. The asynchronous one-sided communication allows the ParVox system effectively overlapping the rendering process and the message passing which is required to resort the data from the rendering stage to the compositing stage.

ParVox is capable of visualizing 3D volume data as a translucent volume with adjustable opacity for each different physical value, or as multiple iso-surfaces at different thresholds and different opacities. It can also slice through the 3D volume and only view a set of slices in either of the three major orthogonal axes. Moreover, it is capa-

ble of animating multiple time-step 3D datasets at any selected viewpoint. ParVox is designed to serve as either an interactive visualization tool for post-processing, or a rendering API (Application Programming Interface) to be linked with any application program. As an distributed visualization system, ParVox provides an X Window based GUI program for display and viewing control, a parallel input library for reading 4D volume datasets in NetCDF format, a network interface program that interfaces with the GUI running on the remote workstation and a parallel wavelet image compression library capable of supporting both lossless and lossy compression. The detail description of the ParVox system and its parallel implementation can be found in [5].

### 6. RESULTS AND DISCUSSION

Various numerical tests have been performed on the 3D code. The results show that the numerical scheme is robust and efficient, and that the general parallel structure allows us to use different partitions to suit various physical domains. Numerical results are obtained for a wide range of Rayleigh number up to  $5 \times 10^7$ . For the steady-state solution of Rayleigh number up to  $10^7$  can be found in [3]. Figure 1 illustrates the velocity field for Rayleigh number  $5 \times 10^7$  in air, with the corresponding temperature field depicted in Figure 2. As the figures illustrate, the flow structure becomes very complicated. Strong transverse flows are generated near the lower and upper corners, and the temperature field shows very thin boundary layers on the two sidewalls and the near-linear temperature stratification in the interior. In the velocity field, separations of flow occur on the bottom near the left corner and on the top near the right corner. Multiple eddies appear in the whole enclosure, and the solution becomes strong convective and time-dependent.

Because of the complexity of the flow structure, visualizing such a data set in detail is a challenge. The main problem is how to capture the dynamics in the velocity field, such as the multiple eddies and flow separations, and high speed near the walls. Those features cover a wide range of velocity values. In lack of a good way to represent the global behavior of vector fields, we choose to use vector magnitude to represent the velocity field. We found, after some experiments, the most effectively way to represent the velocity volume is to use the direct volume with a condensed spectrum color map and an opacity map with mostly low opacity except the values at the lower end and the higher end. The condensed colormap is used to distinguish velocity difference in a small value range. The selected opacity map highlights only the high velocity on the side walls and the low velocity flow in the center of the volume; the less interesting mid-range velocities are mostly transparent, thus revealing the internal low-speed flow. Figure 3 is a snapshot of the ParVox GUI showing the colormap and the opacity map described above and the rendered image based on this specific classification.

In spite of the difficulties associated with large Rayleigh number simulation, our results presented here clearly demonstrate the great potential for applying this approach

to solve high resolution, large Rayleigh number flow in realistic, three-dimensional geometries using parallel systems. Much higher Rayleigh numbers computations of thermal convection in 3D are under investigation. More detailed time-dependent numerical results will be discussed in the full paper.

# References

- [1] S.V. Patankar. Numerical Heat Transfer and Fluid Flow. Hemisphere, New York, 1980.
- [2] S.F. McCormick. Multilevel Adaptive Methods for Partial Differential Equations. Frontiers on Applied Mathematics, SIAM, Philadelphia, 1989.
- [3] P. Wang. Massively parallel finite volume computation of three-dimensional thermal convective flows. Advances in Engineering Software, to appear, 1998.
- [4] L. Westover. Footprint evaluation for volume rendering. Computer Graphics, Proceedings of Siggraph, 24, 1990.
- [5] P. Li and et. al. Parvox a parallel splatting volume rendering system for distributed visualization. *Proceedings of the 1997 IEEE Parallel Rendering Symposium*, pages 7–14, 1997.

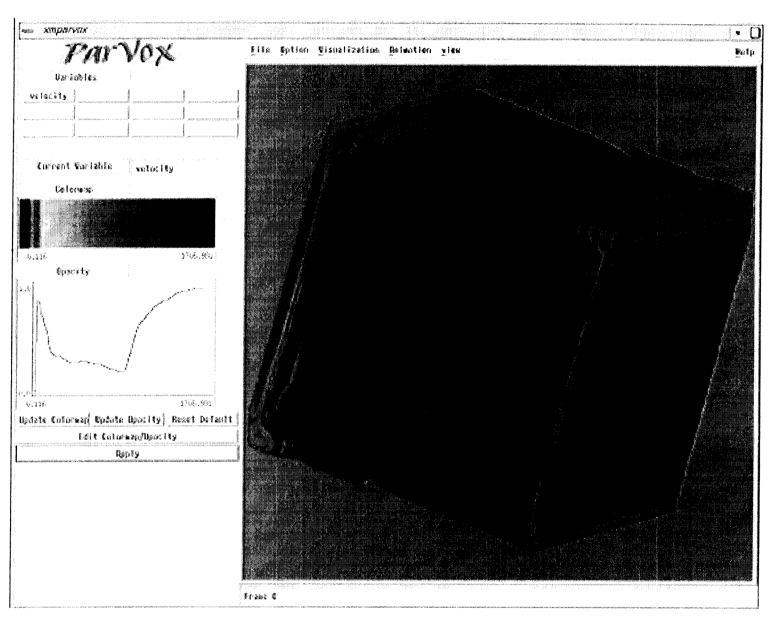


Figure 1: The velocity field for Rayleigh number  $5\times 10^7$  using a computational grid  $128\times 128\times 128$ .

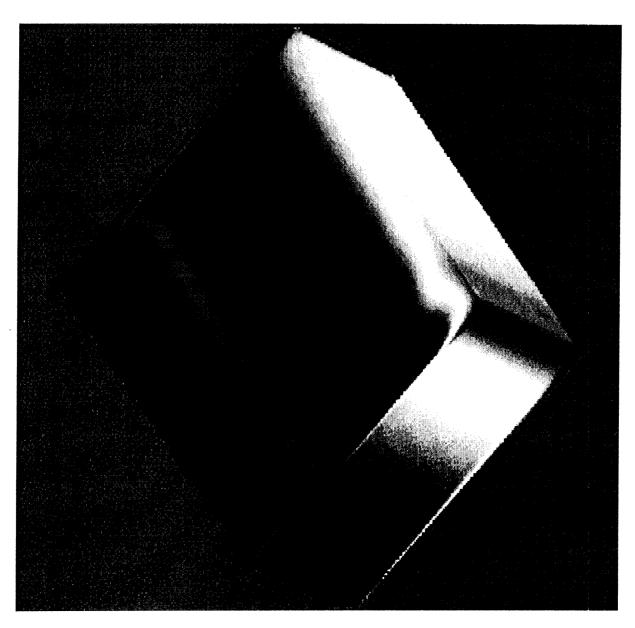


Figure 2: The temperature field for Rayleigh number  $5\times10^7$  using a computational grid  $128\times128\times128$ .